Unification, duality, and de Bruijn graphs

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A question



You forgot your digicode. The keypad lets you enter a sequence of any length. As soon as the correct code appears, the door opens. What sequence do you enter?

The start of an answer

0001AAA0002AA010AAA100AA110011A0012A0021A01110101A101210022A0A0A1A020A01220003AA0211020102A102210A1111200A120112AAA20023 233001320133011300A220302031203A203320A230A130123A113AA131A132A1232113221212A2123311133A1A2A13321A221302131213A21330AA3 0034400414040433314402223022302231142312234443103410042404104210434034004310444014402440430430422040204120442043203 4300441014102410A311340005AA0433030313032303A301A302A303440014201430144011400242024302440214012400A3204420A3304030413042 3A23432243323A323A422244A2A3A24432A3324032413242324A324442AA333340334133422A342334AAA44104510052A0511050105A10521053A05 220502051205A20531054A0452005320541055A015A025A035A0A4A053305030513052305A305420453005430454005510151025103510A411450006 AA05440404140424043404A401A402A403A40455001520153015401550115002520253025402550215012500352035303540355031502250A1503250 23500A4205520A4305530A44050405140524053405A405540A4501350145A115AA151A152A153A154A145211522151215A2153215421453115331513 152315A31543145411544141424143414A412A413A4145511155A125A135A1A4A1552125213521A421553125313531A431554125413541A441504151 41524153415A415550A2503350A35034502450AA50056A0061A060A055512251125513251235113551A2513351A35134511A4512451AA42245A225AA 252A253A254A245322533252325A3254324542254424243424A423A4245522255A235A2A4A255323532A43255423542A44250425142524253425A425 A4355553AA4444504451445244533A453A455AAA5105610062A0611060106A10621063A06220602061206A20631064A06320641065A056200633060306 13062306430642065106640164026403640464065406520563006440604061406240634064406530564006540565006610161026103610461045 11560007AA065505051505250535054505A501A502A503A504A505660016201630164016501660116002620263026402650266021601260036203630 364036503660316022604160326023600462046304640465046604160426033601360426043603460045206620453066304540664045506050615062 50635064506A506650A5601460156A116AA161A162A163A164A165A156211622161216A21632164216521563116331613162316A3164316531564116 44161416241634165416541565116551515251535154515A512A513A514A5156611166A126A136A146A1A5A16621262136214621A521663126313631 4631A5316641264136414641A5416651265136514651A551605161516251635164516A516660A36044602460A460456025603560AA60067A0071A070 A06661226112661326123611366142613361A2614361346114661A36144612461A46145611A56125613561AA52256A226AA262A263A264A265A25632 6424642A542665236524652A552605261526252635264526A5266623362236624362346224662A3624462A46245622A5623562AA53356A336AA363A3 64A365A356433644363436A4365435653365535354535A534A5356633366A346A3A5A366434643A54366534653A553605361536253635364536A5366 546A546664AA555556055615562556355644A564556AAA6106710072A0711070107A10721073A07220702071207A20731074A07320741075A07420751 076A06720073307030713072307A307520761077A017A027A037A047A057A0A6A07530762067300744070407140724073407A407540763067400 7550705071507250735074507450765067600776506760077101710271037104710571046116700084407660606160626063606460656064601460246 03A604A605A60677001720173017401750176017701170027202730274027502760277021701270037203730374037503760377031702270A1703270 237004720473047404750476047704170427033701370A270437034700572057305740575057605770517052705370447014702470A370547045700A 6207720A6307730A6407740A6507750A6607060716072607360746075607A607760A6701570167A117AA171A172A173A174A175A176A167211722171 217A217321742175217621673117331713172317A317431753176316741174417141724173417A417541764167511755171517251735174517A51765 16761176616162616361646165616A612A613A614A615A6167711177A127A137A147A157A1A6A177212721372147215721A621773127313731473157 31A63177412741374147415741A64177512751375147515751A65177612761376147615761A6617061716172617361746175617A617770A470557025 7035704570567026703670467044700784008140804077712271127713271237113771427133714271337142713471147715271537144712471437154714 57115771A471557125713571A57156711A671267136714671AA62267A227AA272A273A274A275A276A267322733272327A3274327532763267422744 2724273427A4275427642675227552725273527452765267652676526766263626362646265626A623A624A625A6267722277A237A247A257A2A6A277323 7324732573246327742374247425742464277523752475257524652776237624762576246662706271627262736274627562746277523372237724372 34799477953794479437954794579957794479557935794579567994679367946794463367433744373437543764367433744373437443734376436 43675337553735374537453765367633766363646365636A634A635A6367733377A347A357A3A6A3774347435743A643775347535753A65377634763 576346637063716372637363746375637463777344733477354734573357734473557345735673346734673446744474447544764475447646754475 5474547A5476546764476646465646A645A6467744477A457A4A6A477545754A65477645764A6647064716472647364746475647A647774557445774 A57456744A674A655A5A565A5A566A556665567A557A557A5576A567655766565666567755577A5A6A57765A66570657165726573657465756574657 775446666706671667266736674667554675667444710781008240811080108410821083408220802081208420831084408320841085408420851086 A08520861087A07820083308030813082308A30843085308620871088A018A028A038A048A058A068A0A7A0863087207830084408040814082408340 844085408640873078400855080508150825083508450845086508740785008660806081608260836084608560846087507860087607870088101810 281038104810581068104711780009440877070717072707370747075707670747014702470347044705470647078800182018301840185018601870 1880118002820283028402850286028702880218012800382038303840385038603870388031802280A1803280238004820483048404850486048704 880418042803380138042804380348005820583058405850586058705880518052805380448014802480438054804580068206830684068506860687 The aim of this talk is to explain what de Bruijn graphs are, and how they relate to a problem of unifiability in logic.

We will see that the connection is made by Stone duality, a general theory for linking syntax and semantics.

Overview

De Bruijn graphs

Unifiability

Stone duality

de Bruijn graphs

The de Bruijn graph $B_d(\Sigma)$ of order d over alphabet Σ is the deterministic automaton that 'remembers the last d letters'.

de Bruijn graphs

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For example, when d = 3 and $\Sigma = \{0, 1\}$:



Named for N. G. de Bruijn (1946), also invented by I. J. Good (1946), implicit in C. Flye Sainte-Marie (1894), and also in ancient Sanskrit prosody; see Knuth, vol. 4A, 7.2.1.7, p. 489.

Graphs

Fix a finite alphabet Σ .

Definition

A Σ -edge-labeled directed graph consists of:

a set of vertices V,

• for each $a \in \Sigma$, an edge relation $\stackrel{a}{\rightarrow} \subseteq V^2$.

Example

For any $d \ge 1$, the de Bruijn graph of order d, $B_d(\Sigma)$, has

• for each
$$a \in \Sigma$$
 and $w \in \Sigma^d$, a labeled edge

$$w \stackrel{a}{\longrightarrow} w'a$$

where w' is the length d-1 suffix of w.

Graph homomorphisms

A homomorphism from a graph G to a graph H is a function from V_G to V_H that preserves labeled edges.



Is there a homomorphism from $B_2(\{0,1\})$ to the graph G?



 $B_2(\{0,1\})$



Is there a homomorphism from $B_2(\{0,1\})$ to the graph G?



Is there a homomorphism from $B_2(\{0,1\})$ to the graph G?



- The codomain graph may fail to be deterministic in general.
- There may be more than one homomorphism.

Is there a homomorphism from $B_2(\{0,1\})$ to the graph H?



Is there a homomorphism from $B_2(\{0,1\})$ to the graph H?



Is there a homomorphism from $B_d(\{0,1\})$ to H for some d?

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Is there a homomorphism from $B_d(\{0,1\})$ to H for some d? How can you be sure that there is none? • Answer for d = 2

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Is there a homomorphism from $B_d(\{0,1\})$ to H for some d? How can you be sure that there is none? • Answer for d = 2How to decide this for a general graph? • Another example The de Bruijn graph mapping problem

Fix a finite alphabet Σ .

Problem (de Bruijn graph mapping)

INPUT. A finite Σ -edge-labeled directed graph G. OUTPUT.

- ▶ a number $d \ge 1$ and a homomorphism $B_d \to G$, or
- 'impossible' if none exists.

We usually consider the *surjective* version of the problem, which is at least as hard. \frown Explanation

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The unifiability problem

We arrived at this problem because of a problem in temporal logic:

Problem (Unifiability in temporal logic of next) INPUT. A formula φ in the temporal logic of next. OUTPUT.

- \blacktriangleright a unifier for φ , or
- 'impossible' if none exists.

Unifiers defined

Two sets: variables $V = \{x, y, ...\}$, constants $C = \{p_1, p_2, ...\}$. A formula is an expression built from these with \lor , \neg , \bot , and **X**. The depth of a formula is the maximum nesting depth of **X** in it. A unifier of a formula $\varphi(x, y, ...)$ is a substitution

$$x \mapsto \sigma_x, y \mapsto \sigma_y, \ldots,$$

where, for each $x \in V$, σ_x is a formula, such that

$$\varphi(\mathbf{x}\mapsto\sigma_{\mathbf{x}},\mathbf{y}\mapsto\sigma_{\mathbf{y}},\dots)$$

is a valid formula.

Validity defined

There are two equivalent views on validity in the logic of next:

Syntactic: A formula φ is valid if the equation φ ≡ ⊤ follows from the rules of Boolean logic, together with

$$\mathbf{X}(\varphi_1 \lor \varphi_2) \equiv \mathbf{X}\varphi_1 \lor \mathbf{X}\varphi_2, \quad \mathbf{X}(\neg \varphi) \equiv \neg \mathbf{X}\varphi, \text{ and } \mathbf{X}\top \equiv \top.$$

Semantic: A formula φ is valid if it evaluates to true in all deterministic transition systems. Xφ is evaluated as 'next φ':



The other connectives are evaluated locally at each state.

The equivalence is called the completeness theorem for the logic.

Is this formula unifiable?

$$[x \leftrightarrow (\neg \mathbf{X} p \land \mathbf{X} \mathbf{X} (x \lor y))] \land [y \leftrightarrow (x \to p)]$$

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Negative example: $x \leftrightarrow \neg \mathbf{X} x$ does not have a unifier.



Unifiers and de Bruijn graph homomorphisms

Let φ be a formula in the logic of next. We construct a finite graph $G(\varphi)$ with edge-labels in $\Sigma := 2^{C}$. (This can take up to $\mathcal{O}(\exp(|\varphi|))$ time.)

Theorem (v.G. & Marti, 2023)

The set of $\equiv\mbox{-classes}$ of depth $\leq d$ ground unifiers of φ is in bijection with

the set of homomorphisms from $B_d(2^C)$ to $G(\varphi)$.

The depth of σ is the maximum depth of the formulas σ_x . A unifier σ is ground if no variables are used in any σ_x . Unifiers σ and σ' are equivalent if $\sigma_x \equiv \sigma'_x$ for all x.











 $G(\varphi)$

The graph $G(\varphi)$ is an image of B_2 , so φ is unifiable.

The more general picture: Equational Unification

Let **V** be a variety of algebras in a signature τ .

Problem (Unifiability in **V**)

Given a finite set E of τ -equations

$$s(x_1,\ldots,x_n)\approx t(x_1,\ldots,x_n)$$
,

does there exist a substitution $x_i \mapsto u_i$ such that

$$\mathbf{V} \models s(u_1,\ldots,u_n) \approx t(u_1,\ldots,u_n)$$

for each $s \approx t$ in E?

Unifiability and finitely presented algebras

Problem (Unifiability in **V**, algebraic version) Given a finitely presented algebra A in **V**, does there exist a homomorphism from A to $F_{\mathbf{V}}(\emptyset)$?

A finite set of τ -equations E gives a finitely presented algebra

$$A := F_{\mathbf{V}}(x_1, \ldots, x_n) / \langle E \rangle_{\mathrm{con}} ,$$

and a ground unifier is a homomorphism $A \to F_{\mathbf{V}}(\emptyset)$.

Idea: It is sometimes useful to dualize this problem and to use stratification of the free algebras.

(Ghilardi 1999, Ghilardi & Zawadowski 2002)

An open problem: Unifiability for modal algebras

A modal algebra is a tuple (B, \Diamond) , with B a Boolean algebra and a function $\Diamond : B \to B$ such that, for all $a, b \in B$,

$$(a \lor b) = (a \lor b) a \lor b and (b \perp = \perp).$$

The unifiability problem for modal algebras is open.

Its unification type is nullary (Jerabek 2011) and slight extensions have undecidable unification (Wolter & Zakharyashev 2006).

Our result: Unifiability for pointed dMA's

A modal algebra (B, \mathbf{X}) is deterministic if, for every $a \in B$,

$$\mathbf{X}\neg a = \neg \mathbf{X}a \; .$$

A pointed dMA is a tuple (B, \mathbf{X}, c) , where (B, \mathbf{X}) is a deterministic modal algebra, and $c \in B$.

Theorem (Marti, v.G., Sweering 2024)

Unifiability is decidable for pointed dMA's.

Proof. Duality + decide the de Bruijn graph mapping problem! Overview

De Bruijn graphs

Unifiability

Stone duality
Stone duality: The big picture



A textbook treatment



M. Gehrke and SvG, Topological Duality for Distributive Lattices, Cambridge University Press (2024).

Boolean algebras

For any set X, the power set $\mathcal{P}(X)$ is a Boolean algebra, i.e., its operations \cup , \cap and ()^c precisely obey the rules of \vee , \wedge , and \neg . Moreover, any function $f: X \to Y$ gives rise to a Boolean algebra homomorphism in the other direction:



Thus, we have a functor $\mathcal{P} \colon \mathsf{Set} \to \mathsf{BA}^{\mathrm{op}}$.

Stone representation for Boolean algebras

Theorem (Stone, 1936)

For any Boolean algebra B, the homomorphism

$$\widehat{(-)} \colon B \hookrightarrow \mathcal{P}(\mathsf{Hom}(B,2)), \quad b \mapsto \{x \colon x(b) = 1\}$$

is injective.

Moreover, the topology τ on spec(B) := Hom(B, 2) generated by the image of $\widehat{(-)}$ is Boolean, and $B \cong \{ \text{ clopens of spec}(B) \}$.

(A Boolean space is a compact space in which any distinct points are separable by a clopen.)

Example

For any set V, spec($\mathbb{F}_{BA}(V)$) is the Cantor space 2^{V} .

Morphisms

Every homomorphism of Boolean algebras arises from a unique continuous function of spaces:



We get an equivalence functor spec: $BA^{op} \rightarrow BoolSp$.

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This is how we find unifiers: A unifier is a Boolean algebra homomorphism, so it must arise from a continuous function!

$$(x_1 \wedge \neg x_2) \vee (x_2 \wedge x_3) \stackrel{?}{=} \top$$

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 $\mathbb{B}(x_1, x_2, x_3)$



$$\underbrace{(x_1 \land \neg x_2) \lor (x_2 \land x_3)}_{\varphi} \stackrel{?}{=} \top$$

 $\mathbb{B}(x_1, x_2, x_3)$















111

Stone duality for (free) pointed dMA's

A pointed dMA is a tuple (B, \mathbf{X}, c) , where $\mathbf{X} \colon B \to B$ is a Boolean homomorphism and $c \in B$.

The dual category consists of tuples (X, f, K) where X is a Boolean space, $f: X \to X$ is continuous, and $K \subseteq X$ is clopen.

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Proposition

Let V a set of variables, $\mathbb{F}(V)$ the free pointed dMA on V. Then

$$\operatorname{spec} \mathbb{F}(V) = (\Sigma^{\omega}, s, \widehat{c}), \quad ext{ where }$$

Σ := 2^{V∪{c}}, the local valuations over variables and constant;
s: Σ^ω → Σ^ω, the shift map, sends (u_i)[∞]_{i=0} to (u_i)[∞]_{i=1},
ĉ = {u ∈ Σ^ω | u₀(c) = 1}.

Unifiers via duality

Let φ be a formula in variables V, WLOG of depth ≤ 1 .

There are bijections between the following sets:

- ► \equiv -classes of ground unifiers of φ of depth $\leq d$;
- ▶ algebra homomorphisms $\sigma : \mathbb{F}(V) \to \mathbb{F}(\emptyset)$ such that each $\sigma(x)$ is of depth $\leq d$ and $\sigma(\varphi) = \top$;
- continuous shift-invariant maps h: 2^ω → Σ^ω with modulus of continuity d, and im(h) ⊆ φ;

• de Bruijn graph homomorphisms $B_d(2) \rightarrow G(\varphi)$.

Admitting homomorphisms from de Bruijn graphs

Theorem (v.G., Marti, Sweering 2024)

For any finite graph $G = (V_G, E_G)$, the following are equivalent:

1. there exist $d \ge 1$ and a surjective homomorphism $B_d(2) \to G$;

2. the graph G is cycle-connected and power-connected.

Condition (2) can be verified in time $\mathcal{O}(\exp(|V_G| + |E_G|))$.

Corollary

Unifiability for pointed dMA's is decidable in 2-exponential time.



▶ De Bruijn graphs can open doors.

Unification problems are hard to solve ad hoc.

Stone duality can give a principled approach for solving them.



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Thank you.

Further references

- van Gool & Marti, Modal unification step by step (2023).
- Gehrke & van Gool, Topological Duality for Distributive Lattices: Theory and Applications (2024).
- Baader & Ghilardi, Unification in Modal and Description Logics (2011).

Credits:

- Digicode picture: Wikipedia, user D4m1en, CC BY-SA 3.0.
- De Bruijn sequence (k = 12, n = 4) generated using code by Joe Sawada.

Appendix

The de Bruijn graphs possess a *very* strong connectedness property, which transfers to images: B_d is power-connected.



Letters read: 011



Letters read: 11



Letters read: 1

The de Bruijn graphs possess a *very* strong connectedness property, which transfers to images: B_d is power-connected. Intuitively: 'You can find out where you are within d steps'.



Letters read:

Let $H = (V_H, E_H)$ be a Σ -graph.

A node u ∈ V_H is a predecessor of a set S ⊆ V_H if, for every letter a ∈ Σ, there exists s ∈ S such that u → s.

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Let $G = (V_G, E_G)$ be a Σ -graph.

▶ The power graph of G is the graph with nodes $\mathcal{P}(V_G)$ and

$$S \stackrel{a}{\rightarrow} T \iff \forall x \in S, \exists y \in T \text{ such that } x \stackrel{a}{\rightarrow}_G y$$
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 .

G is power-connected if, in its power graph, the node V_G is in the closure of the set of nodes {{u} : u ∈ V_G}.

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Cycle-connectedness defined

Let
$$G = (V_G, E_G)$$
 be a Σ -graph and let $w \in \Sigma^+$.
A w-cycle is a path $u \xrightarrow{w} u$, for some $u \in V_G$.

Cycle-connectedness defined

- Let $G = (V_G, E_G)$ be a Σ -graph and let $w \in \Sigma^+$.
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A w-cycle u → u is reachable if there exists k ≥ 1 such that for every node v, there is a path v → u.

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G is cycle-connected if, for every w ∈ Σ⁺, there exists a reachable w-cycle.

Answer for d = 2

▲ Back

The following 'hamburger' graph is not an image of B_2 :



- ▶ In B_2 , we have $00 \xrightarrow{1}{\rightarrow} 01 \xrightarrow{1}{\rightarrow} 11$; a homomorphism must map this to $x \xrightarrow{1}{\rightarrow} z \xrightarrow{1}{\rightarrow} y$.
- Similarly, $11 \xrightarrow{0} 10 \xrightarrow{0} 00$ must go to $y \xrightarrow{0} z \xrightarrow{0} x$.
- But now the edge $10 \xrightarrow{1}{\rightarrow} 01$ is not preserved: $z \xrightarrow{1}{\rightarrow} z$.

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- ▶ In B_2 , we have $00 \xrightarrow{1}{\rightarrow} 01 \xrightarrow{1}{\rightarrow} 11$; a homomorphism must map this to $x \xrightarrow{1}{\rightarrow} z \xrightarrow{1}{\rightarrow} y$.
- Similarly, $11 \xrightarrow{0} 10 \xrightarrow{0} 00$ must go to $y \xrightarrow{0} z \xrightarrow{0} x$.
- But now the edge $10 \xrightarrow{1}{\rightarrow} 01$ is not preserved: $z \xrightarrow{1}{\rightarrow} z$.

Another example

▲ Back

Why can the following 'cone of fries' graph not admit a homomorphism from any B_d ?



(This is a small example of a graph that is 'power-connected' but not 'cycle-connected', witnessing the independence of the two conditions in our characterization of images of de Bruijn graphs.)

Explanation

▲ Back]

Suppose given an oracle for the surjective version. We solve the non-surjective version as follows.

For every subgraph G' of G, use the oracle for the surjective version to decide if a surjective homomorphism onto G' exists. If so, return this homomorphism, now viewed as a homomorphism to G. If the oracle for the surjective version always answers 'impossible', return impossible.

This is correct, because if some homomorphism h to G exists, then it will be surjective onto the subgraph im(h).

Explanation

▲ Back]

Suppose φ were a formula such that $\varphi \leftrightarrow \neg \mathbf{X} \varphi$ is valid.

Consider the transition system with a single state, which is its own successor, and any valuation of the variables.

If φ evaluates to true in this transition system, then $\neg \mathbf{X} \varphi$ evaluates to true as well, by assumption on φ .

But then, by definition of the valuation, $\mathbf{X}\varphi$ should evaluate to false, so φ should evaluate to false.

The other case follows by symmetry.