Math 308R: Bridge to Advanced Mathematics
Homework #7
Due date: Tuesday October 25, 2016, 3:30PM

1. Find and describe an open conjecture in mathematics. Answer the following questions about the conjecture that you choose:
   (a) What is the statement of the conjecture? Describe it in such a way that your fellow students (and your instructor) can understand.
   (b) Who formulated the conjecture, and when?
   (c) Has a prize been offered for a proof of the conjecture?
- Cite any internet and/or library resources that you use.
- Examples of conjectures that you could use are: twin prime conjecture, infinitely many perfect numbers conjecture, Erdős conjecture on arithmetic progressions.

2. Prove or disprove:
   (a) For any integers $a, b$, if $3 \mid ab$ then $3 \mid a$ and $3 \mid b$.
   (b) For any integers $a, b$, if $3 \mid ab$ then $3 \mid a$ or $3 \mid b$.
   (c) For any integers $a, b$, if $4 \mid ab$ then $4 \mid a$ or $4 \mid b$.
   (d) (Extra Credit) For every prime number $p$, for any integers $a, b$, if $p \mid ab$ then $p \mid a$ or $p \mid b$.

3. For each of the following statements, state the negation of the statement, and disprove the statement (i.e., prove the negation):
   (a) For any real numbers $x, y$, $(x + y)^2 = x^2 + y^2$.
   (b) For every integer $x$, there exists an integer $y$ such that $xy \equiv 1 \pmod{4}$.
   (c) There exist integers $p, q$ such that $\sqrt{2} = \frac{p}{q}$.
   (d) There exists a set $A$ such that for every subset $B \subseteq A$, $|A - B| \geq 1$.

4. Let $A = \{1, 2, 3\}$ and let $R$ be the following relation on $A$: $R \overset{\text{def}}{=} \{(1, 2), (2, 2), (3, 2), (3, 3)\}$.
   (a) Draw the graph of the relation $R$.
   (b) Prove or disprove: $R$ is reflexive.
   (c) Prove or disprove: $R$ is symmetric.
   (d) Prove or disprove: $R$ is transitive.
   (e) Draw the graph of the inverse relation $R^{-1}$.
   (f) Using set notation, list the elements of $R^{-1}$.